Tutorial 2

Notation: $G = \mathrm{SL}_2(\mathbb{Q}_p)$, B denote the subgroup of G consisting of upper-triangular matrices, I_1 be the subgroup of $\mathrm{SL}_2(\mathbb{Z}_p)$ consisting of matrices that are upper-triangular unipotent mod p and let T be the subgroup of diagonal matrices.

1. Prove the decomposition:

$$G = BI_1 \sqcup BwI_1,$$

2. Let χ be an $\overline{\mathbb{F}}_p$-valued character of T which is thought of as a character of B using the surjection $B \to T$. Write down a basis for the space $(\mathrm{Ind}_B^G \chi)^{I_1}$.

3. Given a generator T_s in $\mathcal{H}(G, I_1)$, calculate the action of T_s on a function $f \in (\mathrm{Ind}_B^G \chi)^{I_1}$.

4. Use the calculation above to determine when $(\mathrm{Ind}_B^G \chi)^{I_1}$ is simple as an $\mathcal{H}(G, I_1)$-module.